Exam. Code : 103205
Subject Code : 1204

B.A./B.Sc. 5th Semester

MATHEMATICS (Number Theory)

Paper—II

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions from each section.

SECTION—A

1. (a) Prove that 4 does not divide $(m^2 + 2)$ for any integer m.

   (b) Prove that the square of an integer is of the form 3q or 3q + 1 but not of the form 3q + 2, $q \in z$. 5,5

2. (a) If m is an odd integer, show that $8 \mid (m^2 - 1)$.

   (b) If gcd(a, 4) = 2 and gcd(b, 4) = 2, prove that gcd(a + b, 4) = 4. 5,5

3. (a) Use the Euclidean Algorithm to find integers x and y such that gcd(1769, 2378) = 1769x + 2378y.

   (b) Find the general solution (in integers) of the equation 91x + 221y = 1053. 5,5

4. (a) Prove that for each prime $p \geq 5$, $p^2 + 2$ is a composite number.

   (b) If $a \equiv b \pmod{m}$, prove that $a^p \equiv b^p \pmod{m}$ for any positive integer p. 5,5

5. (a) Solve $2x + 7y \equiv 5 \pmod{12}$.

(b) If $x \equiv a \pmod{m}$, prove that either $x \equiv a \pmod{2m}$ or $x \equiv a + m \pmod{2m}$. 5,5

SECTION—B

6. (a) Solve $x \equiv 5 \pmod{11}, x \equiv 14 \pmod{29}$ and $x \equiv 15 \pmod{31}$ by Chinese Remainder Theorem.

(b) If $\gcd(a, 42) = 1$, show that $a^6 \equiv 1 \pmod{168}$. 5,5

7. (a) Prove that an integer $p > 1$ is a prime number iff $\lfloor\underline{p-2} \equiv 1 \pmod{p}$.

(b) Prove that $a^5 \equiv a \pmod{30}$ for all integer a. 5,5

8. (a) Show that $\lfloor\underline{18} = -1 \pmod{437}$, using Wilson's theorem.

(b) Find remainder when $2\lfloor\underline{26}$ is divided by 29. 5,5

9. (a) If $n = 2(2p - 1)$ where p, $2p - 1$ both are prime > 2 then show that $\phi(n + 2) = \phi(n)$, $\phi(n)$ is Euler's phi-function.

(b) Find all positive integers a, b such that :

$$\phi(a, b) = \phi(a) + \phi(b).$$ 5,5

10. (a) Using Euler's theorem, find the last two digits in ordinary decimal representation of 3^{400}.

(b) If x and y are positive real numbers, prove that $[x]\,[y] \leq [xy]$; $[x]$, $[y]$ and $[xy]$ are greatest integer functions. 5,5